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International Journal of Current Research in Life Sciences
Vol. 07, No. 03, pp.1289-1296, March, 2018

ISSN: 2319-9490

RESEARCH ARTICLE

OPTIMAL MANAGEMENT OF PREY–PREDATOR COMMUNITY WITH RATIO–DEPENDENT FUNCTIONAL RESPONSE

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Received 09th January, 2018; Accepted 20th February, 2018; Published Online 30th March, 2018

ABSTRACT

In the paper a non-linear mathematical model is proposed and analyzed to discuss the combined harvesting policy for a prey predator community with ratio dependent functional response. Criteria for local stability, instability and global stability of the non-negative equilibria are obtained. In last an optimal harvesting to harvest prey-predator species is derived.

Key words: prey-predator, ratio-dependent response, stability, optimal harvesting.

Citation: Dr. Pragya Mishra and Manju Agarwal. 2018. "Optimal management of prey–predator community with ratio–dependent functional response" *International Journal of Current Research in Life Sciences*, 7, (03), 1289-1296.

INTRODUCTION

Almost in all prey–predator system considered in literature, predator response function is taken to be a function of prey population only. This may be a tenable assumption for an experimental situation where a single predator searches for prey in an area, but in real life it is likely that individual predators will interact with each other. Leaving aside, the issue of predator cooperation in hunting and subduing prey, it is likely that predator encounter will lead to competitive interactions. Intra–specific competitive interactions between individual predator can affect their birth and death rates and also the predator's efficiency in finding and killing prey i.e. predator functional response. If predators encounter each other at a rate 'b', encounter prey at a rate 'a' and each encounter between predators results in wasted time 'w', then these assumptions lead to the formula given by Beddington (1975)

$$f(x,y) = \frac{ax}{1 + bw'y},$$

where x and y are prey and predator densities respectively.

Hassell and Varley (1969), using reparameterization w = w'b/a suggests the following form of functional response

$$f(x,y) = \frac{ax}{1 + awy}.$$

Again, using the logic that predators waste time in handling prey and dealing with other predators, that are added together to reduce the amount of time left for search. This logic leads to the following form for combined functional response

$$f(x,y) = \frac{ax}{1 + awy + ahx},$$

where h is the handling time.

This functional form was first proposed by Beddington (1975) and independently by De Angleis et.al. (1975).

Now, assuming awy + ahx >>> 1 (where predator interference is very strong), we have

$$f(x,y) \approx \frac{ax}{awy + ahx} = \frac{\alpha x}{\beta y + x} = \frac{\alpha x / y}{\beta + x / y},$$

which is the Holling type II functional response, but with the ratio (x/ y) replacing prey density x. This form is known as 'Ratio–dependent functional response'. It is clear from the above form that predator density should have a strong effect on predator functional response. Many biological systems with following features can produce such predator dependence:

- Group hunting by the predator,
- Facultative and costly antipredator defense by the prey,
- Density dependent and time consuming social interactions between predators,
- Aggressive interactions between searching predators that encounter each other, and
- A limited number of high quality sites where predator captures prey rapidly.

Recently, there has been little work on Ratio–dependent theory, which can be roughly stated as that the per capita predator growth rate should be function of ratio of prey to predator abundance. This theory is strongly supported by numerous field and laboratory experiments and observations done by Arditi and Ginzburg [1] and Arditi et.al. [2]. Generally, a ratio dependent prey–predator model takes the following form:

$$\dot{x}(t) = xf(x) - yp(x/y),$$
$$\dot{y}(t) = (\pi p(x/y) - d)y, \quad x(0) > 0, \ y(0) > 0,$$

with $\lim_{(x,y)\to(0,0)} yp(x/y) = 0.$

When $p(x) = \dfrac{\alpha x}{\beta + x}$ and $f(x) = r\left(1 - \dfrac{x}{K}\right)$, this becomes a ratio dependent prey–predator model with Michaelis–Menten function response.

This system was studied in detail by Hsu et.al. [5], Hsu and Hwang [4], and others. Geometrically the difference of prey–dependent model and ratio dependent model are obvious, the former has a vertical predator isocline, while the later has a slanted one passing through the origin. Arditi and Ginzburg [1], Berryman [3] showed that the ratio–dependent models are capable of producing richer and more reasonable or acceptable dynamics. It has been also shown [6] that the ratio–dependent type models do not produce the so–called paradox of enrichment, which states that according to the Lotka–Vottera type predator–prey theory (with Michaelis–Menten Holling type functional response) enriching a predator–prey system will cause an increase in the equilibrium density of the predator but not in that of the prey, and will destabilize the community equilibrium. Another similar paradox is the so called "biological control paradox" which states that according to the classical predator–prey model, we cannot have both a low and stable prey equilibrium density, is also no longer valid for ratio–dependent systems.

Kuang and Beretta [7] did qualitative analysis of global behaviors of solutions of a ratio–dependent predator–prey system. Hsu et.al. [5] discussed a three tropic ratio–dependent food chain model and its application in biological control process. They also provide scenarios when biological control is feasible and when it may fail. Keeping above facts in mind, in this paper, we propose and analyze a non–linear mathematical model of prey–predator community having ratio–dependent functional response with harvesting effort. The organization of paper is as follows: section 2 deals with mathematical model and some basic results. In section 3, existence of equilibria and their stability is discussed. An optimal harvesting policy is obtained in section 4, numerical calculation is done in section 5 and finally, a discussion is presented in section 6.

The Mathematical Model

We consider a prey–predator community where predation is governed by ratio–dependent functional response. It is assumed that the dynamics of prey population follows logistic model and is subjected to a dynamic harvesting. To maintain desired level of population, the regulatory agency imposes a tax s > 0 (negative value of s denotes subsidy) per unit biomass of the landed prey and predator population. Taking note of above, we propose a system of differential equations for model as follows:

$$\frac{dx}{dt} = rx(1 - x/K) - \frac{\alpha xy}{\beta y + x} - q_1 Ex = F_1(x,y,E),$$
$$\frac{dy}{dt} = \frac{\pi \alpha xy}{\beta y + x} - dy - q_2 Ey = F_2(x,y,E),$$
$$\frac{dE}{dt} = \alpha_0 E[(p_1 - s)q_1 x + (p_2 - s)q_2 y - c] = F_3(x,y,E) \qquad (1)$$

with $x(0) > 0, \quad y(0) > 0, \quad E(0) > 0$.

The functions $F_i(x, y, z)$, i = 1, 2, 3 are defined for x > 0, y > 0, E ≥ 0 and $\lim_{(x,y,E)\to 0} F_i(x, y, z) = 0$.

Here x(t) and y(t) are the concentrations of prey and predator population respectively and E(t) is the combined effort applied to harvest both prey and predator population at time t > 0. 'r' and 'K' are intrinsic growth rate coefficient and carrying capacity of logistically growing prey population. 'α' is the constant uptake rate coefficient of predator at which it consumes prey and 'β' is the intraspecific competitive coefficient among predators. 'π' (0 < π < 1) denotes the fraction of predation term that contributes in predator's growth and'd' is the death rate coefficient for predator. q_1 and q_2 are the constant catchability coefficients for prey and predator population, 'p_1' and 'p_2' are the fixed price per unit of prey and predator population respectively, and 'c' is the fixed cost of harvesting population per unit of effort. The constant α_0 is called stiffness parameter measuring the strength of reaction of effort to the perceived rent. Next there is a theorem for persistence and boundedness of solutions.

Theorem 2.1: All solutions of the system (1) with initial conditions are non–negative and bounded.

Proof: From system of eqs. (1), we have

$$\left.\frac{dx}{dt}\right|_{x=0} = 0, \left.\frac{dy}{dt}\right|_{y=0} = 0, \left.\frac{dE}{dt}\right|_{E=0} = 0$$

Since initially x(t), y(t) and E(t) are all positive and non–zero, therefore, from above condition and continuity of system we conclude that all solutions of system (1) are non–negative.

Again from (1), on integration we get,

$$x(t) \le \frac{K}{\left[1 + \frac{x(0) - K}{x(0)} e^{-rt}\right]},$$

as $t \to \infty \quad x(t) \le K$.

Now, we consider, $\mu = \alpha_0(p_1 - s)\,x + \alpha_0(p_2 - s)\,y + E$, thentaking $p \ge \max(p_1, p_2)$, and

$$\dot{\mu} + \in \mu \le \alpha_0(p - s)\left[(r+\in)x - \frac{rx^2}{K}\right] + (\in - \alpha_0 c)E + (\in - d)y,$$

choosing $\in < \min(\alpha_0 c, d)$, on integrating and taking limit as $t\to\infty$ we get

$$\mu \le \frac{\alpha_0(p - s)(r+\in)^2 K}{4r\in}.$$

Hence, theorem is proved.

The Mathematical Analysis

It is easy to check that there exist five equilibrium points in which existence of trivial axial equilibrium $P_0(0,0,0)$ and axial equilibrium $P_1(K,0,0)$ are obvious. Other equilibrium points are

$$P_2(\bar{x}, \bar{y}, 0), P_3(\bar{\bar{x}}, 0, \bar{\bar{E}}) \text{ and } P^*(x^*, y^*, E^*),$$

$$\bar{x} = \frac{K}{r}\left[r - \frac{(\pi\alpha - d)}{\pi\beta}\right], \qquad \bar{y} = \frac{(\pi\alpha - d)\bar{x}}{\beta d}$$

and

$$\bar{\bar{x}} = \frac{c}{(p_1 - s)q_1}, \qquad \bar{\bar{E}} = \frac{r}{q_1}\left(1 - \frac{\bar{\bar{x}}}{K}\right).$$

So, P_2 exists when $\pi r\beta > \pi\alpha - d > 0$, which also implies $\alpha > d/\pi$.The condition for existence of P_3 is $\bar{\bar{x}} < K$, which on simplification gives

$$s < p_1 - \frac{c}{q_1 K}. \qquad (2)$$

The above expression gives the upper bound for regulatory tax in the absence of predator.

Existence of P*(x*,y*,E*) : x*, y* and E* are the positive solutions of following algebraic equations

$$y = \frac{c-(p_1-s)q_1x}{(p_2-s)q_2}.$$

$$E = \frac{1}{q_1}\left[r(1-x/K) - \frac{\alpha y}{\beta y + x}\right].$$

Eliminating y from above equations, we get

$$E = \frac{1}{q_1}\left[r\left(1-\frac{x}{K}\right) - \frac{\alpha\{c-(p_1-s)q_1x\}}{\beta c + \{(p_2-s)q_2 - \beta(p_1-s)q_1\}x}\right]. \qquad (3)$$

As $x \to 0$, $E \to \frac{1}{q_1}\left(r - \frac{\alpha}{\beta}\right) = E_1 > 0$ when $r\beta > \alpha$,

(which is a necessary condition for the existence of ($\bar{x}, \bar{y}, 0$)).

When $x \to K$, $E \to \dfrac{-\alpha\{c-(p_1-s)q_1K\}}{\beta c + \{(p_2-s)q_2 - \beta(p_1-s)q_1\}K} = E_2 < 0,$

and also

$$\frac{dE}{dx} = \frac{1}{q_1}\left[-\frac{r}{K} + \frac{\alpha c}{(\beta y + x)^2(p_2-s)q_2}\right] < 0$$

$$for\ \frac{r}{K} > \frac{\alpha c}{(\beta y + x)^2(p_2-s)q_2}.$$

$E \to 0$, when $x \to x_1$, where x_1 is the positive roots of following quadratic equation: $a_1x^2 + a_2x + a_3 = 0$,

$$a_1 = \left\{\frac{\beta(p_1-s)q_1}{(p_2-s)q_2} - 1\right\}\frac{r}{K}, \quad a_2 = r - \left\{\frac{(r\beta-\alpha)(p_1-s)q_1K + r\beta c}{K(p_2-s)q_2}\right\},$$

$$a_3 = \frac{(r\beta-\alpha)c}{(p_2-s)q_2}.$$

Again on eliminating y, we have

$$E = \frac{1}{q_2}\left[\frac{\pi\alpha x(p_2-s)q_2}{\beta c + \{(p_2-s)q_2 - \beta(p_1-s)q_1\}x} - d\right], \qquad (4)$$

when $x \to 0$, $E \to -d/q_2 < 0$ and $E \to 0$, when $x \to x_2$ where

$$x_2 = \frac{\beta c}{(\pi\alpha/d - 1)(p_2-s)q_2 + (p_1-s)q_1\beta} > 0 \quad \text{when } \pi\alpha > d$$

Also

$$\frac{dE}{dx} = \frac{\pi\alpha\beta c}{q_2^2(\beta y + x)^2(p_2-s)} > 0$$

So, the two isoclines given by eqs. (3) and (4) intersect at unique point (x*, E*) when condition (2) holds along with inequalities $d < \pi\alpha < \pi r\beta$ and $x_2 < x_1$.

Also for the existence of P*, we must also have

$$y^* = \frac{c-(p_1-s)q_1x^*}{(p_2-s)q_2} > 0 \quad \text{or } s > p_1 - \frac{c}{q_1x^*}.$$

The characteristic equation corresponding to P*(x*,y*,E*) can be written as

$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0,$

where

$$b_1 = x^*\left(\frac{r}{K} - \frac{\alpha y^*}{(\beta y^* + x^*)^2}\right) + \frac{\pi\alpha\beta x^* y^*}{(\beta y^* + x^*)^2},$$

$$b_2 = \frac{\pi\alpha\beta x^{*2} y^*}{(\beta y^* + x^*)^2}\left\{\frac{r}{K} - \frac{\beta y^*}{(\beta y^* + x^*)^2} + \frac{\alpha y^*}{(\beta y^* + x^*)^2}\right\} + \alpha_0 q_1^2 (p_1 - s) x^* E^* + \alpha_0 q_2^2 (p_2 - s) y^* E^*,$$

$$b_3 = \alpha_0 q_2^2 (p_2 - s) x^* y^* E^* \left\{\frac{r}{K} - \frac{\alpha y^*}{(\beta y^* + x^*)^2}\right\} - \frac{\alpha\alpha_0 q_1 q_2 (p_1 - s) x^{*2} y^* E^*}{(\beta y^* + x^*)^2}$$
$$+ \frac{q_1 \pi\alpha\beta\alpha_0 E^* x^* y^*}{(\beta y^* + x^*)^2}\{(p_2 - s) q_2 y^* + (p_1 - s) q_1 x^*\}.$$

So, when $\frac{r}{K} > \frac{\alpha y^*}{(\beta y^* + x^*)^2}$, $b_3 > 0$ and $b_1 b_2 - b_3 > 0$, then by Routh Hurwitz criteria, all roots of characteristic equation have negative real parts and P* is locally asymptotically stable equilibrium point.

For global stability, we consider positive definite function V(x,y,E) as

$$V = x - x^* - x^* - \ell n \frac{x}{x^*} + k_1\left(y - y^* - y^* \ell n \frac{y}{y^*}\right) + k_2\left(E - E^* - E^* \ell n \frac{E}{E^*}\right)$$

where k_1, k_2 are positive constants to be chosen suitably.

Differentiating V with respect to time t along the solutions of system (1), we get

$$\dot{V} = (x - x^*)\left[r\left(1 - \frac{x}{K}\right) - \frac{\alpha y}{\beta y + x} - q_1 E\right] + k_1 (y - y^*)\left[\frac{\pi\alpha x}{\beta y + x} - d - q_2 E\right] + k_2\alpha_0(E - E^*)[(p_1 - s)q_1 x + (p_2 - s)q_2 y - c]$$

Using equations giving interior equilibrium point P*(x*,y*,E*) and after some algebraic manipulations, and choosing $k_1 = \frac{(p_2 - s)}{(p_1 - s)}$ and $k_2 = \frac{1}{\alpha_0 (p_1 - s)}$, above quadratic equation becomes

$$\dot{V} = -\left\{\frac{r}{K} - \frac{\alpha y^*}{(\beta y^* + x^*)(\beta y + x)}\right\}(x - x^*)^2 - \frac{(p_2 - s)\pi\alpha\beta x^* (y - y^*)^2}{(p_1 - s)(\beta y^* + x^*)(\beta y + x)}$$
$$+ \left\{-\alpha x^* + \frac{(p_2 - s)}{(p_1 - s)}\pi\alpha\beta y^*\right\}\frac{(y - y^*)(x - x^*)}{(\beta y^* + x^*)(\beta y + x)}$$

So, $\dot{V}$ is negative definite when

$$\left\{-\alpha x^* + \frac{(p_2 - s)}{(p_1 - s)}\pi\alpha\beta y^*\right\}^2 < \frac{4\pi\alpha\beta(p_2 - s)x^*}{(p_1 - s)}\left\{\frac{r}{K} - \frac{\alpha y^*}{(\beta y^* + x^*)(\beta y + x)}\right\}(\beta y^* + x^*)(\beta y + x),$$

which holds only when

$$\left\{x^* - \frac{(p_2 - s)}{(p_1 - s)}\pi\beta y^*\right\}^2 < \frac{4\pi\beta(p_2 - s)x^*}{\alpha(p_1 - s)}\left\{(\beta y^* + x^*)\frac{(\beta y + x)r}{K} - \alpha y^*\right\}.$$

Also, the subset S of Ω such that $S = \{(x, y, E) \in \overline{\Omega} : \dot{V} = 0\}$

The largest invariant set in this is $\{(x, y, E) \in \overline{\Omega}, x = x^*$, y= y* and E = E*}. Hence by LaSalle's invariance principle, P* is globally asymptotically stable with respect to all solutions initiating in the interior of the positive orthant.

Optimal Harvesting Policy

Now, we derive the optimal policy that maximizes the net revenue to the society that is given by

$\pi(x, y, E, s, t) = (p_1q_1x + p_2q_2y - c)E.$

Thus, our objective is to solve the optimization problem:

$$\max \int_0^{\infty} e^{-\delta t}(p_1q_1x + p_2q_2y - c)E\,dt,$$

where δ subject to state equations of (1) and to the control constraint,

$$s_{min} \le s \le s_{max}, \qquad (5)$$

is the instantaneous annual rate of discount.

To solve the above problem, we use Pontryagin's Maximum Principle. The associated Hamiltonian function is given by

$$H(x,y,E,s,t) = e^{-\delta t}(p_1q_1x + p_2q_2y - c)E + \lambda_1(t)\left[rx\left(1-\frac{x}{K}\right) - \frac{\alpha xy}{(\beta y + x)} - q_1Ex\right] + \lambda_2(t)\left[\frac{\pi\alpha xy}{(\beta y + x)} - dy - q_2Ey\right]$$
$$+ \lambda_3(t)\alpha_0E[(p_1 - s)q_1x + (p_2 - s)q_2y - c],$$

where $\lambda_1, \lambda_2, \lambda_3$ are adjoint variables.

For 'H' to be maximum on the control set (5), we must have

$$\frac{\partial H}{\partial \tau} = 0, \text{ which implies } \lambda_3(t) = 0. \qquad (6)$$

Now, from the maximum principle, we must have

$$\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial x}, \quad \frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial y}, \quad \frac{d\lambda_3}{dt} = -\frac{\partial H}{\partial E}.$$

On rewriting above equations, we have

$$\frac{d\lambda_1}{dt} = -e^{-\delta t}p_1q_1E - \lambda_1(t)\left[r\left(1-\frac{2x}{K}\right) - \frac{\alpha y}{\beta y + x} + \frac{\alpha xy}{(\beta y + x)^2} - q_1E\right] - \lambda_2(t)\frac{\pi\alpha\beta y^2}{(\beta y + x)^2}, \qquad (7)$$

$$\frac{d\lambda_2}{dt} = -e^{-\delta t}p_2q_2E + \lambda_1(t)\frac{\alpha x^2}{(\beta y + x)^2} - \lambda_2(t)\left[\frac{\pi\alpha x}{\beta y + x} - \frac{\pi\alpha xy\beta}{(\beta y + x)^2} - d - q_2E\right], \qquad (8)$$

$$\frac{d\lambda_3}{dt} = -e^{-\delta t}(p_1q_1x + p_2q_2y - c) + \lambda_1q_1x + \lambda_2q_2y = 0. \qquad (9)$$

From eqs. (6) and (9), we get $\lambda_1q_1x + \lambda_2q_2y = e^{-\delta t}(p_1q_1x + p_2q_2y - c).$ (10)

Now, considering interior equilibrium point P* and substituting the value of λ_2 from eq. (10), we get a differential equation

$$\frac{d\lambda_1}{dt} - A_1\lambda_1 = -A_2e^{-\delta t}.$$

A solution of above differential equation is given by

$$\lambda_1(t) = \frac{A_2}{A_1 + \delta}e^{-\delta t} \qquad (11)$$

where

$$A_1 = x^*\left\{\frac{r}{K} - \frac{\alpha y^*}{(\beta y^* + x^*)^2}\right\} + \frac{\pi\alpha\beta q_1 x^* y^*}{q_2(\beta y^* + x^*)^2},$$

$$A_2 = p_1 q_1 E^* + \frac{\pi\alpha\beta y^*(p_1 q_1 x^* + p_2 q_2 y^* - c)}{q_2(\beta y^* + x^*)^2}.$$

Similarly we have

$$\lambda_2(t) = \frac{B_2}{B_1 + \delta} e^{-\delta t}, \qquad (12)$$

Where

$$B_1 = \frac{\pi\alpha\beta x^* y^*}{(\beta y^* + x^*)^2},$$

$$B_2 = p_2 q_2 E^* - \frac{\alpha x^{*2} A_2}{(\beta y^* + x^*)^2 (A_1 + \delta)}.$$

Now, from eq. (10), on substituting the values of λ_1 and λ_2, we get

$$q_1 x^*\left(p_1 - \frac{A_2}{A_1 + \delta}\right) + q_2 y^*\left(p_2 - \frac{B_2}{B_1 + \delta}\right) = c. \qquad (13)$$

For an optimal effort, we have

$$y = \frac{c - (p_1 - s)q_1 x}{(p_2 - s)q_2}. \qquad (14)$$

Above two equations give the optimal equilibrium levels of prey–predator population i.e. $x^* = x_\delta, y^* = y_\delta$

Then the optimal equilibrium levels of effort and tax are given by

$$E_\delta = \frac{1}{q_1}\left[r\left(1 - \frac{x_\delta}{K}\right) - \frac{\alpha y_\delta}{\beta y_\delta + x_\delta}\right]$$

$$s_\delta = \frac{p_1 q_1 x_\delta + p_2 q_2 y_\delta - c}{q_1 x_\delta + q_2 y_\delta}$$

Equation (10) can be written as

$$\lambda_1 q_1 x^* + \lambda_2 q_2 y^* - c = q_1 x^* \frac{A_2}{A_1 + \delta} + q_2 y^* \frac{B_2}{B_1 + \delta} \quad \to 0 \quad \text{as } \delta \to \infty$$

Hence, the net economic revenue is zero when discounting factor is infinitely large.

Numerical Example

Let us consider the following hypothetical values of different parameters as below:

$r = 2$	$\beta = 1$	$\pi = 0.5$
$K = 7$	$d = 0.1$	$q_1 = 0.1$
$\alpha = 1$	$\delta = 0.1$	$q_2 = 0.1$
$p_1 = 10$	$p_2 = 8$	$c = 6$

Now for different values of tax, we have the following results:

Table 1. Equilibrium level of prey population and arresting effect for different values of tax

s	x*	y*	E*
0	5.1274	1.8981	3.6491
2	4.8193	3.8855	1.7683
4	4.5405	6.9728	0.9718
6	4.2879	12.8560	0.2506
8	4.0587	30.0000	–0.4042

From above table, we see that the equilibrium level of prey population x* and effort E* decreases while the corresponding level of predator population y* increases as the tax increases. There exists a value of tax ($6 < s < 8$) imposed by the regulatory agency, for which the equilibrium effort level becomes zero and in this case prey–predator population remain unexploited. It can also be verified here that for above equilibrium values, all stability conditions hold and hence P*(x*,y*,E*) is stable equilibrium point for the parameter values taken above. Also the optimal equilibrium level of prey–predator population, harvesting effort and tax are obtained as

$x_\delta = 5.5975$, $y_\delta = 0.0089$,
$E_\delta = 3.9921$, $s_\delta = -0.7306$.

Here, negative value of tax shows that regulatory agency should provide subsidies to maintain the optimal equilibrium level of population.

DISCUSSION

In the present paper, we have proposed and analyzed a nonlinear mathematical model to study the dynamics of a prey–predator community with ratio–depedent functional response and dynamic harvesting effort with tax as a control instrument to avoid the over exploitation of population. Ratio dependent functional form presents a real situation where predation is affected by both prey and predator population. We have shown persistence and boundedness of solutions. We have proved the existence of equilibrium points under certain conditions. Interior equilibrium point P* is defined and stable under certain parametric conditions. The stability of the system implies that prey–predator population and harvesting effort settle down to their respective equilibrium level under certain conditions. Using Pontryagin’s Maximum principle, an optimal policy to harvest prey–predator population with ratio–dependent functional response has been discussed and optimal equilibrium levels of prey–predator population, effort and tax have been obtained. It has been shown that the total user’s cost of harvest per unit effort is equal to the present value of marginal revenue of effort at the optimal equilibrium level. It has also been noted that increase in discount rate decreases the economic rent and even it may tend to zero if the discount rate tends to infinity.

Acknowledgement: Author Dr. Pragya Mishra is thankful to CSIR for SRF during the research work.

REFERENCES

Arditi, R. and Ginzburg, L.R. 1989. Coupling in predator–prey dynamics: ratio–dependence., *J. Theoretical Biology*, 139, 311–326.
Arditi, R., Ginzburg, L.R. and Akchakya, H.R. 1991. Variation in Plankton densities among lakes: a case of ratio–dependent models, American Naturalist 138, 1287–1296.
Berryman, A.A. 1992. The origins and evolution of predator–prey theory, Ecology 73., 1530–1535.
Hsu, S.B. and Huang, T.W. 1995. Global stability for a class of predator–prey system, *SIAM J. Appl. Math*. 55, 763–783.
Hsu, S.B. Huang, T.W. and Kuang, Y. 2003. A ratio dependent food chain model and its applications to biological control, Math. Biosci. 181, 55–83.
Kuang, Y. 1999. Rich dynamics of Gause–type ratio–dependent predator–prey systems, Fields institute communications, 21, 325–337.
Kuang, Y. and E. Beretta, 1998. Global qualitative analysis of a ratio–dependent predator–prey system, *J. Math. Biol.* 636, 389–406.

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