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RESEARCH ARTICLE

RELATIONS ON BASIC OPERATIONS IN INTUITIONISTIC FUZZY SETS

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ABSTRACT

In this paper, various operations in Intuitionistic Fuzzy Sets are discussed. Some theorems are proved for establishing the properties of intuitionistic fuzzy operators with respect to different Intuitionistic fuzzy sets.

Key words: Intuitionistic Fuzzy Set (IFS) and Intuitionistic fuzzy set operators, relations and basic operations

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1. INTRODUCTION

L.A. Zadeh [5] introduced the notion of a Fuzzy sub set μ of a Set X as a function from X to [0,1]. After the introduction of Fuzzy sets by L.A. Zadeh [5], the Fuzzy concept has been introduced in almost all branches of Mathematics. Then the concept of Intuitionistic Fuzzy Set (IFS) was introduced by Atanassov [1] as a generalization of the notation of a Fuzzy set.

2. PRELIMINARIES

Definition 2.1 - Crisp Sets: Either the element belongs to the set or it does not is known as Crisp set.

Definition 2.2 - Fuzzy set:

A fuzzy set is built from a reference set called universe of discourse. Assume that U = $\{x_1, x_2, ..., x_n\}$ is the universe of discourse, then a fuzzy set A in U (A ⊂ U) is defined as a set of ordered pairs

$\{(x_i, \mu_A(xi))\}$

Where $x_i \in U$, $\mu_A : U \to [0, 1]$ is the membership function of A and, $\mu_A(x) \in [0, 1]$ is the degree of membership of x in A.

Definition 2.3 -Fuzzy Set Operations:

(i)Union : The membership function of the union of two fuzzy sets A & B with membership functions and it is defined as the maximum the two individual membership functions. This is called the maximum criterion.

$\mu_{AUB} = \max(\mu_A, \mu_B)$

(ii) ***Intersection:*** The membership function of the intersection of two fuzzy sets A & B with membership functions and it is defined as the minimum of two individual membership functions. This is called the minimum criterion.

$\boldsymbol{\mu_{A\cap B} = \min(\mu_A, \mu_B)}$

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(iii) ***Complement:*** The membership function of the complement of a fuzzy set A with membership function is defined as the negation of the specified membership function. This is called the negation criterion.

$$\boldsymbol{\mu_{\bar{A}} = 1 - \mu_A}$$

Definition 2.4 - Intuitionistic Fuzzy Set: An Intuitionistic Fuzzy Set A in a non empty set X is an object having the form A= { $\langle x, \mu_A(x), \nu_A(x)\rangle | x\epsilon E\rangle$ } where the functions μ_A : X→ [0.1] and ν_A : X→ [0.1] denote the degrees of membership and non membership of the element xϵX to A respectively and satisfy $0 \le \mu_A(x) + \nu_A(x) \le 1$ for all xϵX. The family of all intuitionistic fuzzy sets in X denoted by IFS (X).

Definition 2.5 - Basic Operations and Relations over Intuitionistic Fuzzy Sets: Two IFSs A and B the following operations and relations can be defined as

$A \subseteq B$ *iff* For every $(\forall x \in E)(\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x))$ …………………………………..(2.1)

$A \supseteq B$ *iff* $B \subseteq A$ …………………………………..(2.2)

$A = B$ *iff* $(\forall x \in E)(\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x))$ ………………………………….(2.3)

$A \cap B = \{ \langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle | x\epsilon E\}$ ………………………………….(2.4)

$A \cup B = \{ \langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle | x\epsilon E\}$ ………………………………….(2.5)

$A+B = \{ \langle x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x)\rangle | x\epsilon E\}$ ………………………………….(2.6)

$A.B = \{ \langle x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)\rangle | x\epsilon E\}$ ………………………………...(2.7)

$A@B = \left\{\langle x, \frac{\mu_{A(x)+\mu_B(x)}}{2}, \frac{\nu_{A(x)+\nu_B(x)}}{2}\rangle \,|x\epsilon E\right\}$ ………………………………...(2.8)

Definition 2.6 - Special intuitionistic fuzzy sets:

$O^* = \{\langle x, 0, 1\rangle | x\epsilon E\}$ ………………………………….(2.9)

$E^* = \{\langle x, 1, 0\rangle | x\epsilon E\}$ ………………………………..(2.10)

$U^* = \{\langle x, 0, 0\rangle | x\epsilon E\}$ ………………………………..(2.11)

P (E^*) = {A\A = $\{\langle x, \mu_A(x), \nu_A(x)\rangle | x\epsilon E\}$} ………………………………..(2.12)

P (U^*) = {B\B = $\{\langle x, 0, \nu_B(x)\rangle | x\epsilon E\}$} ………………………………..(2.13)

P (O^*) = $\{O^*\}$ ……………………………….(2.14)

3. Relations On Basic Operations In Intuitionistic Fuzzy Sets

For every two given IFSs A and B has the forms:
A-B = $\{\langle x, \mu_{A-B}(x), \nu_{A-B}(x)\rangle | x\epsilon E\}$ ………………………………….(3.1)

Where

$$\mu_{A-B}(x) = \begin{cases} \dfrac{\mu_A(x) - \mu_B(x)}{1 - \mu_B(x)} & if \mu_A(x) \ge \mu_B(x) and \nu_A(x) \le \nu_B(x) and \nu_B(x) \succ 0 \\ 0, otherwise \end{cases}$$ ………………………………….(3.2)

$$\nu_{A-B}(x) = \begin{cases} \dfrac{\nu_A(x)}{\nu_B(x)} & if \mu_A(x) \ge \mu_B(x) and \nu_A(x) \le \nu_B(x) and \nu_B(x) > 0 \\ 0, otherwise \end{cases}$$ ………………………………….(3.3)

and

A:B = $\{\langle x, \mu_{A:B}(x), \nu_{A:B}(x)\rangle | x\epsilon E\}$ ………………………………….(3.4)

where

$$\mu_{A:B}(x) = \begin{cases} \dfrac{\mu_A(x)}{\mu_B(x)} & if \mu_A(x) \le \mu_B(x) and \nu_A(x) \ge \nu_B(x) and \mu_B(x) > 0 \\ 0, otherwise \end{cases} \qquad \text{……………………………………..(3.5)}$$

and

$$\nu_{A:B}(x) = \begin{cases} \dfrac{\nu_A(x) - \nu_B(x)}{1 - \nu_B(x)} & if \mu_A(x) \le \mu_B(x) and \nu_A(x) \ge \nu_B(x) and \mu_B(x) > 0 \\ 1, otherwise \end{cases} \qquad \text{……………………………………(3.6)}$$

Theorem 3.1

Prove that A-A=O*

Proof:

Let A = $\{\langle x, \mu_A(x), \nu_A(x)\rangle | x\epsilon E\}$, we have
$\mu_{A-A}(x) = 0$ and $\nu_{A-A}(x) = 1$
A-A = $\{\langle x, 0, 1\rangle | x\epsilon E\rangle\}$ = O*
Hence it proved.

Theorem 3.2

Prove that A: A=E*

Proof:
Let A = $\{\langle x, \mu_A(x), \nu_A(x)\rangle | x\epsilon E\}$, we have
$\mu_{A:A}(x) = 1$ and $\nu_{A:A}(x) = 0$
A: A = $\{\langle x, 1, 0\rangle | x\epsilon E\}$ = E*
Hence it proved.

Theorem 3.3

Prove that A-O*=A

Proof:
Let A = $\{\langle x, \mu_A(x), \nu_A(x)\rangle | x\epsilon E\}$, we have
$\mu_{A-O*}(x) = \mu_A(x)$ and $\nu_{A-O*}(x) = \nu_A(x)$
A-O* = $\{\langle x, \mu_A(x), \nu_A(x)\rangle | x\epsilon E\rangle\}$ = A
Hence it proved.

Theorem 3.4

Prove that A: E*=A

Proof:

Let A = $\{\langle x, \mu_A(x), \nu_A(x)\rangle | x\epsilon E\}$, we have
$\mu_{A:E*}(x) = \mu_A(x)$ and $\nu_{A:E*}(x) = \nu_A(x)$
A: E* = $\{\langle x, 1, 0\rangle | x\epsilon E\rangle\}$ = A
Hence it proved.

Theorem 3.5

Prove that A-U*=O*

Proof:
Let A = $\{\langle x, \mu_A(x), \nu_A(x)\rangle | x\epsilon E\}$, we have
$\mu_{A-U*}(x) = 0$ and $\nu_{A-U*}(x) = 1$
A-U* = $\{\langle x, 0, 1\rangle | x\epsilon E\}$ = O*

Hence it proved.

Theorem 3.6

Prove that A: U*=O*

Proof:

Let A = $\{\langle x, \mu_A(x), \nu_A(x)\rangle | x\epsilon E\}$, we have

$\mu_{A:U*}(x) = 0$ and $\nu_{A:U*}(x) = 1$

A: U* = $\{\langle x, 0, 1\rangle | x\epsilon E\}$ = O*

Hence it proved.

Theorem 3.7

Prove that (A-B) +B=A

Proof:

Let A-B = $\{\langle x, \mu_{A-B}(x), \nu_{A-B}(x)\rangle | x\epsilon E\}$, we have

$$\mu_{A-B}(x) = \frac{\mu_A(x) - \mu_B(x)}{1-\mu_B(x)} \quad \text{and} \quad \nu_{A-B}(x) = \frac{\nu_A(x)}{\nu_B(x)}$$

$$\text{A-B} = \{\langle x, \frac{\mu_A(x) - \mu_B(x)}{1-\mu_B(x)}, \frac{\nu_A(x)}{\nu_B(x)} \rangle | x \in E\}$$

$$\text{(A-B)+B} = \{\langle x, \frac{\mu_A(x) - \mu_B(x)}{1-\mu_B(x)}, \frac{\nu_A(x)}{\nu_B(x)} \rangle | x \in E\} + \{\langle x, \mu_B(x), \nu_B(x) \rangle | x \in E\}$$

$$= \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in E\}$$

(A-B)+B = A

Hence it proved.

Theorem 3.8

Prove that (A: B) B=A

Proof:

Let A: B = $\{\langle x, \mu_{A:B}(x), \nu_{A:B}(x)\rangle | x\epsilon E\}$, we have

$$\mu_{A:B}(x) = \frac{\mu_A(x)}{\mu_B(x)} \quad \text{and} \quad \nu_{A:B}(x) = \frac{\nu_A(x) - \nu_B(x)}{1-\nu_B(x)}$$

$$\text{A: B} = \{\langle x, \frac{\mu_A(x)}{\mu_B(x)}, \frac{\nu_A(x) - \nu_B(x)}{1-\nu_B(x)} \rangle | x \in E\}$$

$$\text{(A: B)B} = \{\langle x, \frac{\mu_A(x)}{\mu_B(x)}, \frac{\nu_A(x) - \nu_B(x)}{1-\nu_B(x)} \rangle | x \in E\} \{\langle x, \mu_B(x), \nu_B(x) \rangle | x \in E\}$$

$$= \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in E\}$$

(A-B)+B = A

Hence it proved.

Theorem 3.9

Prove that (A-B)-C= (A-C)-B

Proof:

L.H.S:

Let A-B = $\{\langle x, \mu_{A-B}(x), \nu_{A-B}(x)\rangle | x\epsilon E\}$, we have

$$\mu_{A-B}(x) = \frac{\mu_A(x) - \mu_B(x)}{1-\mu_B(x)} \quad \text{and} \quad \nu_{A-B}(x) = \frac{\nu_A(x)}{\nu_B(x)}$$

A-B $=\{\langle x, \frac{\mu_A(x)-\mu_B(x)}{1-\mu_B(x)}, \frac{\nu_A(x)}{\nu_B(x)} \rangle | x \in E\}$

(A-B)-C $=\{\langle x, \frac{\mu_A(x)-\mu_B(x)}{1-\mu_B(x)}, \frac{\nu_A(x)}{\nu_B(x)} \rangle | x \in E\}$-$\{\langle x, \mu_C(x), \nu_C(x) \rangle | x \in E\}$

$$= \{\left\langle x, \frac{\mu_A(x)-\mu_B(x)-\mu_C(x)+\mu_B(x)\mu_C(x)}{1-\mu_B(x)}, \frac{\nu_A(x)}{\nu_B(x)\nu_C(x)} \right\rangle | x \in E\} \quad \text{………………………..(I)}$$

R.H.S:

Let A-C = $\{\langle x, \mu_{A-C}(x), \nu_{A-C}(x)\rangle | x\epsilon E\}$, we have

$$\mu_{A-C}(x) = \frac{\mu_A(x)-\mu_C(x)}{1-\mu_C(x)} \quad \text{and} \quad \nu_{A-C}(x) = \frac{\nu_A(x)}{\nu_C(x)}$$

A-C $=\{\langle x, \frac{\mu_A(x)-\mu_C(x)}{1-\mu_C(x)}, \frac{\nu_A(x)}{\nu_C(x)} \rangle | x \in E\}$

(A-C)-B$=\{\langle x, \frac{\mu_A(x)-\mu_C(x)}{1-\mu_C(x)}, \frac{\nu_A(x)}{\nu_C(x)} \rangle | x \in E\}$-$\{\langle x, \mu_B(x), \nu_B(x) \rangle | x \in E\}$

$$= \{\left\langle x, \frac{\mu_A(x)-\mu_B(x)-\mu_C(x)+\mu_B(x)\mu_C(x)}{1-\mu_B(x)}, \frac{\nu_A(x)}{\nu_B(x)\nu_C(x)} \right\rangle | x \in E\} \quad \text{………………………..(II)}$$

From (I) & (II) L.H.S=R.H.S
Hence it proved.

Theorem 3.10

Prove that (A: B): C= (A: C): B

Proof:
L.H.S:

Let A: B = $\{\langle x, \mu_{A:B}(x), \nu_{A:B}(x)\rangle | x\epsilon E\}$, we have

$$\mu_{A:B}(x) = \frac{\mu_A(x)}{\mu_B(x)} \quad \text{and} \quad \nu_{A:B}(x) = \frac{\nu_A(x)-\nu_B(x)}{1-\nu_B(x)}$$

(A: B): C $=\{\langle x, \frac{\mu_A(x)}{\mu_B(x)}, \frac{\nu_A(x)-\nu_B(x)}{1-\nu_B(x)} \rangle | x \in E\} : \{\langle x, \mu_C(x), \nu_C(x) \rangle | x \in E\}$

$$=\left\{\left\langle x, \frac{\mu_A(x)}{\mu_B(x)\mu_C(x)}, \frac{\nu_A(x)-\nu_B(x)-\nu_C(x)-\nu_B(x)\nu_C(x)}{(1-\nu_C(x))(1-\nu_B(x))} \right\rangle | x \in E\right\} \quad \text{…………………………(III)}$$

R.H.S

Let A: C = $\{\langle x, \mu_{A:C}(x), \nu_{A:C}(x)\rangle | x\epsilon E\}$, we have

$$\mu_{A:C}(x) = \frac{\mu_A(x)}{\mu_C(x)} \quad \text{and} \quad \nu_{A:C}(x) = \frac{\nu_A(x)-\nu_C(x)}{1-\nu_c(x)}$$

(A: C): B $=\{\langle x, \frac{\mu_A(x)}{\mu_c(x)}, \frac{\nu_A(x)-\nu_c(x)}{1-\nu_c(x)} \rangle | x \in E\} : \{\langle x, \mu_B(x), \nu_B(x) \rangle | x \in E\}$

$$=\left\{\left\langle x, \frac{\mu_A(x)}{\mu_B(x)\mu_C(x)}, \frac{\nu_A(x)-\nu_B(x)-\nu_C(x)-\nu_B(x)\nu_C(x)}{(1-\nu_C(x))(1-\nu_B(x))} \right\rangle | x \in E\right\} \quad \text{…………………………..(IV)}$$

From (III) & (IV) L.H.S =R.H.S
Hence it proved.

Conclusion

We have defined different operations of Intuitionistic Fuzzy Sets. Using these, we have proved different relations between these operators in the intuitionistic fuzzy sets.

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