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RESEARCH ARTICLE

SET THEORETICAL OPERATIONS IN INTUITIONISTIC FUZZY SETS

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ABSTRACT

In this paper, various operations in Intuitionistic Fuzzy Sets are discussed. Some theorems are proved for establishing the properties of Intuitionistic Fuzzy operators with respect to different Intuitionistic Fuzzy sets.

Key words: Intuitionistic Fuzzy Set (IFS) and Intuitionistic Fuzzy Set operators.

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1. INTRODUCTION

L.A. Zadeh [5] introduced the notion of a Fuzzy sub set μ of a Set X as a function from X to [0,1]. After the introduction of Fuzzy sets by L.A.Zadeh [5], the Fuzzy concept has been introduced in almost all branches of Mathematics. Then the concept of Intuitionistic Fuzzy Set (IFS) was introduced by Atanassov [1] as a generalization of the notation of a Fuzzy set.

2. PRELIMINARIES

Definition 2.1- Crisp sets: The crisp set is either an element belongs to the set or it does not.

Definition 2.2- Fuzzy Set: Let X is a nonempty set. A fuzzy set A in X is characterized by its membership function μ_A: X → [0, 1] and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each x ∈ X.

Definition 2.3 - Fuzzy Set Operations:

(i) ***Union :*** The membership function of the union of two fuzzy sets A & B with membership functions and it is defined as the maximum the two individual membership functions. This is called the maximum criterion.

$\boldsymbol{\mu_{AUB} = \max(\mu_A, \mu_B)}$

(ii) ***Intersection:*** The membership function of the intersection of two fuzzy sets A & B with membership functions and it is defined as the minimum of two individual membership functions. This is called the minimum criterion.

$\boldsymbol{\mu_{AUB} = \min(\mu_A, \mu_B)}$

(iii) Complement: The membership function of the complement of a fuzzy set A with membership function is defined as the negation of the specified membership function. This is called the negation criterion.

$\boldsymbol{\mu_{\bar{A}} = 1 - \mu_A}$

Definition 2.4 - Intuitionistic Fuzzy sets: Intuitionistic fuzzy sets are sets whose elements have degrees of membership and non-membership. Intuitionistic fuzzy sets have been introduced by Kashmir Atanassov (1983) as an extension of Lotfi Zadeh's notion of fuzzy set, which itself extends the classical notion of a set. An Intuitionistic Fuzzy Set A in a non empty set X is an object having the form A = $\{\langle x, \mu_A(x), \nu_A(x) | x \epsilon E\rangle\}$ where the functions μ_A : X→[0.1] and $\nu_A(x)$: X→[0.1] denote the degrees of membership and non membership of the element xϵX to A respectively and satisfy 0≤ μ(x)+ $\nu_A(x)$ ≤ 1 for al l xϵX. The family of all intuitionistic fuzzy sets in X denoted by IFS (X).

Definition 2.5 - Basic Operations and Relations over Intuitionistic Fuzzy Sets:

$A \subseteq B$ iff $(\forall x \in E)(\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x))$ ……………………………………..(2.1)

$A \supseteq B$ iff $B \subseteq A$ ……………………………………..(2.2)

$A = B$ iff $(\forall x \in E)(\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x))$ …………………………………….(2.3)

$A \cap B = \{ \langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle | x \epsilon E \}$ ……………………………………(2.4)

$A \cup B = \{ \langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle | x \epsilon E \}$ …………………………………….(2.5)

$A+B = \{ \langle x, \mu_A(x)+\mu_B(x)-\mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x) \rangle | x \epsilon E \}$ …………………………………….(2.6)

$A.B = \{ \langle x, \mu_A(x)\mu_B(x), \nu_A(x)+\nu_B(x)-\nu_A(x)\nu_B(x) \rangle | x \epsilon E \}$ ……………………………………(2.7)

$A@B = \left\{ \langle x, \frac{\mu_{A(x)}+\mu_{B(x)}}{2}, \frac{\nu_{A(x)}+\nu_{B(x)}}{2} \rangle \,|\, x \epsilon E \right\}$ …………………………………….(2.8)

Definition 2.6 - Special intuitionistic fuzzy

$O^* = \{ \langle x, 0, 1 \rangle | x \epsilon E \}$ ……………………………………(2.9)

$E^* = \{ \langle x, 1, 0 \rangle | x \epsilon E \}$ …………………………………...(2.10)

$U^* = \{ \langle x, 0, 0 \rangle | x \epsilon E \}$ …………………………………...(2.11)

$P(E^*) = \{ A \backslash A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle | x \epsilon E \} \}$ …………………………………..(2.12)

$P(U^*) = \{ B \backslash B = \{ \langle x, 0, \nu_B(x) \rangle | x \epsilon E \} \}$ …………………………………(2.13)

$P(O^*) = \{ O^* \}$ …………………………………(2.14)

3. SET THEORETICAL SUBTRACTION PROPERTIES IN INTUITIONISTIC FUZZY SETS

For every two given IFSs A and B has the form

$$A \,|\, B = \left\{ \left\langle \min(\mu_A(x), \nu_B(x)), \max(\mu_B(x), \nu\ (x)) \right\rangle \,|\, x \in E \right\}$$

Theorem 3.1

Prove that $A|E^* = O^*$

Proof:

Let $E^* = \{ \langle x, 1, 0 \rangle | x \epsilon E \}$

We have $A|E^* = \left\{ \left\langle \min(\mu_A(x), 0), \max(\mu_B(x), 1) \right\rangle \,|\, x \in E \right\}$

$= \left\{ \langle 0, 1 \rangle \,|\, x \in E \right\}$

$A|E^* = \left\{ \langle x, 0, 1 \rangle \,|\, x \in E \right\} = O^*$

Hence it completes the proof.

Theorem 3.2

Prove that $A|O^* = A$

Proof:

Let $O^* = \{ \langle x, 0, 1 \rangle | x \epsilon E \}$

We have $A|O^* = \left\{ \left\langle \min(\mu_A(x), 1), \max(0, \nu_B(x)) \right\rangle \,|\, x \in E \right\}$

$= \left\{ \langle \mu_A(x), \nu_A(x) \rangle \,|\, x \in E \right\}$

$A|O^* = A$

Hence it completes the proof.

Theorem 3.3

Prove that E*|A = $\overline{A}$

Proof:

E*|A = $\left\{\left\langle \min(1, \nu_A(x)), \max(0, \mu_A(x))\right\rangle \mid x \in E\right\}$

= $\left\{\left\langle \nu_A(x), \mu_A(x)\right\rangle \mid x \in E\right\}$

= $\overline{A}$

Hence it completes the proof.

Theorem 3.4

Prove that O*| A= O*

Proof:

Let O*= $\{\langle x, 0, 1\rangle | x \epsilon E\}$

We have O*|A = $\left\{\left\langle \min(0, \nu_A(x)), \max(1, \mu_A(x))\right\rangle \mid x \in E\right\}$

= $\left\{\left\langle 0,1\right\rangle \mid x \in E\right\}$

O*|A = O*

Hence it completes the proof.

Theorem 3.5

Prove that $\overline{A \mid B} = \overline{A} \cup B$

Proof:

$A \mid B = \left\{\left\langle \min\left(\mu_A(x), \nu_B(x)\right), \max\left(\mu_B(x), \nu\ (x)\right)\right\rangle \mid x \in E\right\}$ …………………………..(1)

Case (i) If $\mu_B(x) < \nu_A(x)$

$(1) \Rightarrow \left\{\left\langle \min\left(\mu_A(x), \nu\ (x)\right), \nu_A(x)\right\rangle \mid x \in E\right\}$

$\leq \mu_A(x) + \nu_A(x) \leq 1$

Case (ii) If $\mu_B(x) > \nu_A(x)$

$(1) \Rightarrow \left\{\left\langle \min\left(\mu_A(x), \nu\ (x)\right), \mu_B(x)\right\rangle \mid x \in E\right\}$

$\leq \nu_B(x) + \mu_B(x) \leq 1$

$\overline{A \mid B} = \left\{\left\langle \max\left(\mu_B(x), \nu_A(x)\right), \min\left(\mu_A(x), \nu_B(x)\right)\right\rangle \mid x \in E\right\}$ by …………………………..(1)

In both cases (i) & (ii)

$\overline{A \mid B} \leq 1$

$\overline{A} \cup B = \left\{\left\langle x, \nu_A(x), \mu_A(x)\right\rangle \mid x \in E\right\} \cup \left\{\left\langle x, \mu_B(x), \nu_B(x)\right\rangle \mid x \in E\right\}$

$= \left\{\left\langle x, \max\left\{\nu_A(x), \mu_B(x)\right\}, \min\left\{\mu_A(x), \nu_B(x)\right\}\right\rangle \mid x \in E\right\}$ …………………………..(2)

$\therefore$ By case (i)

$(2) \Rightarrow \left\{\left\langle x, \mu_A(x) + \min\left\{\mu_A(x), \nu_B(x)\right\}\right\rangle x \in E\right\}$

$\leq \nu_B(x) + \nu_B(x) = 1$

$\overline{A} \bigcup B \leq 1$

By case (ii)

$$(2) \Rightarrow \{\langle x, \mu_B(x), \min\{\mu_A(x), \nu_B(x)\}\rangle x \in E\}$$

$$\leq \mu_B(x) + \mu_A(x) = 1$$

$\overline{A} \bigcup B \leq 1$

$\therefore$ (1)=(2) in both cases

$\therefore$ $\overline{A | B} = \overline{A} \bigcup B$

Hence it completes the proof.

Theorem 3.5

Prove that

(a)E*|E*=O* (e) O*|U*=O*
(b)U*|E*=O* (f) E*|O*=E*
(c)O*|E*=O* (g) U*|O*=U*
(d) U*|U*=U* (h) O*|O*=O*

Proof:

Let $O^* = \{\langle x, 0, 1\rangle | x \epsilon E\}$ ………………………...(3.1)

$E^* = \{\langle x, 1, 0\rangle | x \epsilon E\}$ ………………………...(3.2)

$U^* = \{\langle x, 0, 0\rangle | x \epsilon E\}$ …………………………(3.3)

(a)E*|E = $\{\langle x, \min(1,0), \max(0,1)\rangle\}$

= $\{\langle x, 0, 1\rangle\}$ =O*

(b)U*|E*= $\{\langle x, \min(0,0), \max(0,1)\rangle\}$

= $\{\langle x, 0, 1\rangle\}$ =O*

(c)O*|E*= $\{\langle x, \min(0,0), \max(1,0)\rangle\}$

= $\{\langle x, 0, 1\rangle\}$ =O*

(d) U*|U*= $\{\langle x, \min(0,0), \max(0,0)\rangle\}$

= $\{\langle x, 0, 0\rangle\}$ =U*

(e) O*|U*= $\{\langle x, \min(0,0), \max(0,1)\rangle\}$

= $\{\langle x, 0, 1\rangle\}$ =O*

(f) E*|O*= $\{\langle x, \min(1,1), \max(0,0)\rangle\}$

= $\{\langle x, 1, 0\rangle\}$ =E*

(g)U*|O*= $\{\langle x, \min(0,1), \max(0,0)\rangle\}$

= $\{\langle x, 0, 0\rangle\}$ =O*

(h) O*|O*= $\{\langle x, \min(0,1), \max(1,0)\rangle\}$
= $\{\langle x,0,1\rangle\}$ =O*

Hence it completes the proof.

Conclusion

We have defined different operations of Intuitionistic Fuzzy Sets. Using these, we have proved different relations between these operators in the intuitionistic fuzzy sets.

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