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RESEARCH ARTICLE

GENERAL REDUCED SECOND ZAGREB INDEX OF CERTAIN NETWORKS

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ABSTRACT

A topological index is a numeric quantity from structural graph of a molecule. In this paper, we introduce the general reduced second Zagreb index of a graph. We compute the reduced second Zagreb index, reduced second hyper-Zagreb index and general reduced second Zagreb index of hexagonal, oxide, rhombus oxide and honeycomb networks.

Key words: Reduced second Zagreb index, Reduced second hyper-Zagreb index, General reduced second Zagreb index, Network.

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INTRODUCTION

In this paper, we consider finite simple connected graphs. Let G be a graph with a vertex set $V(G)$ and an edge set $E(G)$. Let $d_G(v)$ denote the degree of a vertex v in G which is the number of vertices adjacent to v. Any undefined term here may be found in Kulli [1]. A molecular graph is a finite, simple graph such that its vertices correspond to the atoms and the edges to the bonds. Topological indices are useful for establishing correlation between the structure of a molecular compound and its physico-chemical properties. Numerous topological indices have been considered in Theoretical Chemistry and have found some applications, especially in QSPR/QSAR research.

Recently Furtula et al. in [2] proposed the reduced second Zagreb index, defined as

$$RM_2(G) = \sum_{uv\in E(G)} \left(d_G(u)-1\right)\left(d_G(v)-1\right).$$

Considering the reduced second Zagreb index, Kulli in [3] defined the reduced second Zagreb polynomial as

$$RM_2(G,x) = \sum_{uv\in E(G)} x^{\left(d_G(u)-1\right)\left(d_G(v)-1\right)}. \qquad \text{.......................................(1)}$$

In [3], Kulli introduced the reduced second hyper-Zagreb index, defined as

$$RHM_2(G) = \sum_{uv\in E(G)} \left[\left(d_G(u)-1\right)\left(d_G(v)-1\right)\right]^2.$$

Considering the reduced second hyper-Zagreb index, Kulli in [3] defined the reduced second hyper-Zagreb polynomial of a graph G as

$$RHM_2(G,x) = \sum_{uv\in E(G)} x^{\left[\left(d_G(u)-1\right)\left(d_G(v)-1\right)\right]^2}. \qquad \text{.......................................(2)}$$

We now introduce the general reduced second Zagreb index of a graph, defined as

$$RM_2^a(G) = \sum_{uv\in E(G)} \left[\left(d_G(u)-1\right)\left(d_G(v)-1\right)\right]^a \qquad \text{.......................................(3)}$$

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where a is a real number.

Recently, some new reduced indices were studied, for example, in [4, 5, 6, 7, 8]. Also some new topological indices were studied, for example, in [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19].

In this paper, the reduced second Zagreb and reduced second hyper-Zagreb indices and their polynomials of hexagonal, oxide, rhombus oxide and honeycomb networks are determined. For more information about networks see [20].

Results for Hexagonal Networks

It is known that there exist three regular plane tilings with composition of some kind of regular polygons such as triangular, hexagonal and square. Triangular tiling is used in the construction of hexagonal networks. This network is symbolized by HX_n, where n is the number of vertices in each side of hexagon. A hexagonal network of dimension six is presented in Figure 1.

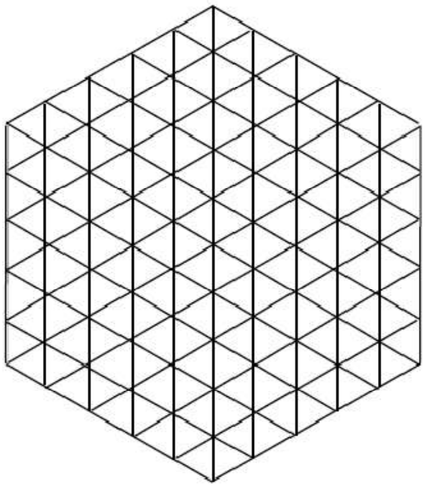

Figure 1. Hexagonal network of dimension six

Let G be the graph of a hexagonal network HX_n with $3n^2–3n+1$ vertices and $9n^2–15n+6$ edges. By calculation in HX_n, there are five types of edges based on the degree of end vertices of each edge as given in Table 1.

Table 1. Edge partition of HX_n

$d_G(u), d_G(v) \backslash uv \in E(G)$	(3, 4)	(3, 6)	(4, 4)	(4, 6)	(6, 6)
Number of edges	12	6	$6n – 18$	$12n – 24$	$9n^2 – 33n + 30$

In the following theorem, we compute the general reduced second Zagreb index of HX_n.

Theorem 1. The general reduced second Zagreb index of a hexagonal network HX_n is

$$RH_2^a(HX_n) = 6^a \times 12 + 10^a \times 6 + 9^a(6n – 18) + 15^a(12n – 24) + 25^a(9n^2 – 33n + 30) . \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Proof: Let $G=HX_n$ be the graph of hexagonal network. By using equation (3) and Table 1, we deduce

$$RM_2^a(HX_n) = \sum_{uv \in E(G)} \left[(d_G(u)-1)(d_G(v)-1) \right]^a$$

$= [(3–1)(4–1)]^a 12+[(3–1)(6–1)]^a 6+[(4–1)(4–1)]^a(6n–18) +[(4–1)(6–1)]^a(12n–24)+[(6–1)(6–1)]^a(9n^2–33n–30)$
$= 6^a \times 12 + 10^a \times 6 + 9^a(6n – 18) + 15^a(12n – 24) + 25^a(9n^2 – 33n + 30)$

We obtain the following results by Theorem 1.

Corollary 1.1. The reduced second Zagreb index of HX_n is
$RM_2(HX_n) = 225n^2 – 591n + 360$.

Proof: Put $a = 1$ in equation (4), we get the desired result.

Corollary 1.2. The reduced second hyper-Zagreb index of HX_n is
$RHM_2(HX_n) = 5625n^2 – 17439n + 12924$.

Proof: Put $a = 2$ in equation (4), we get the desired result.

Theorem 2. The reduced second Zagreb polynomial of HX_n is

$$RH_2(HX_n, x) = 12x^6 + 6x^{10} + (6n - 8)x^9 + (12n - 24)x^{15} + (9n^2 - 33n + 30)x^{25}.$$

Proof: Let $G = HX_n$ be the graph of hexagonal network. By using equation (1) and Table 1, we deduce

$$RM_2(HX_n, x) = \sum_{uv \in E(G)} x^{(d_G(u)-1)(d_G(v)-1)}$$

$$= 12x^6 + 6x^{10} + (6n - 8)\, x^9 + (12n - 24)\, x^{15} + (9n^2 - 33n + 30)\, x^{25}.$$

Theorem 3. The reduced second hyper-Zagreb polynomial of HX_n is

$$RHM_2(HX_n, x) = 12x^{36} + (6n - 8)x^{81} + 6x^{100} + (12n - 18)x^{225} + (9n^2 - 33n + 30)x^{625}.$$

Proof: Let $G = HX_n$ be the graph of hexagonal network. By using equation (2) and Table 1, we deduce

$$RHM_2(HX_n, x) = \sum_{uv \in E(G)} x^{[(d_G(u)-1)(d_G(v)-1)]^2}$$

$$= 12x^{36} + 6x^{100} + (6n - 8)\, x^{81} + (12n - 18)\, x^{225} + (9n^2 - 33n + 30)\, x^{625}.$$

Oxide Networks

Oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension n is denoted by OX_n. An oxide network of dimension five is shown in Figure-2.

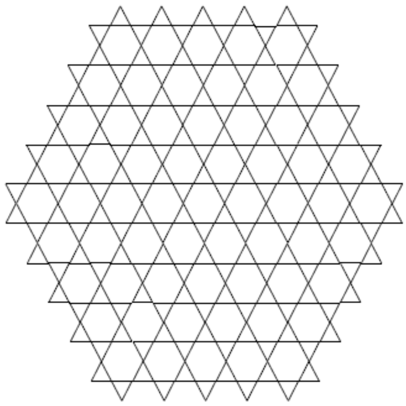

Figure 2. Oxide network of dimension five

Let G be the graph of an oxide network OX_n with $9n^2+3n$ vertices and $18n^2$ edges. By calculation, in OX_n, there are two types of edges based on the degree of end vertices of each edge as given in Table 2.

Table 2. Edge partition of OX_n

$d_G(u), d_G(v) \backslash uv \in E(G)$	(2, 4)	(4, 4)
Number of edges	$12n$	$18n^2 - 12n$

In the following theorem, we compute the general reduced second Zagreb index of OX_n.

Theorem 4. The general reduced second Zagreb index of an oxide network OX_n is

$$RM_2^a(OX_n) = 3^a \times 12n + 9^a(18n^2 - 12n). \qquad \text{………………………………………(5)}$$

Proof: Let $G = OX_n$ be the graph of oxide network. By using equation (3) and Table 2, we deduce

$$RM_2^a(OX_n) = \sum_{uv \in E(G)} \left[(d_G(u)-1)(d_G(v)-1)\right]^a$$

$= [(2–1)(4–1)]^a 12n + [(4–1)(4–1)]^a (18n^2 – 12n)$

$= 3^a \times 12n + 9^a(18n^2 – 12n)$

We obtain the following results by Theorem 4.

Corollary 4.1. The reduced second Zagreb index of OX_n is

$$RM_2(OX_n) = 162n^2 – 72n.$$

Proof: Put $a = 1$ in equation (5), we get the desired result.

Corollary 4.2. The reduced second hyper-Zagreb index of OX_n is

$$RHM_2(OX_n) = 1458n^2 – 864n.$$

Proof: Put $a = 2$ in equation (5), we get the desired result.

Theorem 5. The reduced second Zagreb polynomial of OX_n is

$$RM_2(OX_n, x) = 12nx^3 + (18n^2 – 12n)\, x^9.$$

Proof: Let $G=OX_n$ be the graph of oxide network. By using equation (1) and Table 2, we obtain

$$RM_2(OX_n, x) = \sum_{uv\in E(G)} x^{(d_G(u)-1)(d_G(v)-1)}$$

$$= 12nx^{(2–1)(4–1)} + (18n^2 – 12n)^{(4–1)(4–1)}$$
$$= 12nx^3 + (18n^2 – 12n)x^9.$$

Theorem 6. The reduced second hyper - Zagreb polynomial of OX_n is

$$RHM_2(OX_n, x) = 12nx^9 + (18n^2 – 12n)\, x^{81}.$$

Proof: Let $G=OX_n$ be the graph of oxide network. By using equation (2) and Table 2, we derive

$$RHM_2(OX_n, x) = \sum_{uv\in E(G)} x^{[(d_G(u)-1)(d_G(v)-1)]^2}$$

$$= 12nx^{[(2-1)(4-1)]^2} + (18n^2 - 12n)x^{[(4-1)(4-1)]^2}$$
$$= 12nx^9 + (18n^2 – 12n)x^{81}.$$

Rhombus Oxide Networks

A rhombus oxide network of dimension n is denoted by $RHOX_n$. A 3- dimensional rhombus oxide network is shown in Figure 3.

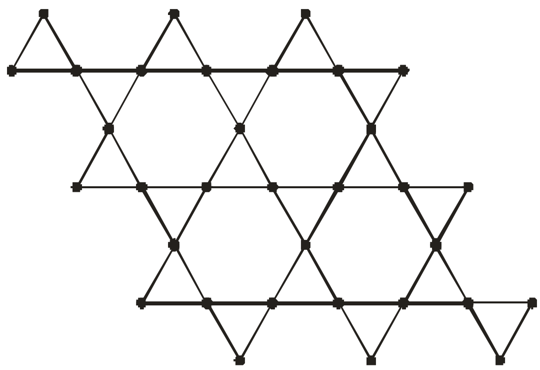

Figure 3. Rhombus oxide network of dimension 3

Let G be the graph of a rhombus oxide network $RHOX_n$ with $3n^2+2n$ vertices and $6n^2$ edges. By calculation, in $RHOX_n$, there are three types of edges based on the degree of end vertices of each edge as given in Table 3.

Table 3. Edge partition of $RHOX_n$

$d_G(u), d_G(v)\backslash uv \in E(G)$	(2, 2)	(2, 4)	(4, 4)
Number of edges	2	$8n – 4$	$6n^2 – 8n + 2$

In the following theorem, we determine the general reduced second Zagreb index of $RHOX_n$.

Theorem 7. The general reduced second Zagreb index of a rhombus oxide network $RHOX_n$ is

$$RM_2^a(RHOX_n) = 2 + 3^a(8n-4) + 9^a(6n^2-8n+2). \quad \ldots\ldots(6)$$

Proof: Let $G=RHOX_n$ be the graph of rhombus oxide network. By using equation (3) and Table 3, we deduce

$$RM_2^a(RHOX_n) = \sum_{uv\in E(G)} \left[(d_G(u)-1)(d_G(v)-1)\right]^a$$
$$= [(2-1)(2-1)]^a 2 + [(2-1)(4-1)]^a(8n-4) + [(4-1)(4-1)]^a(6n^2-8n+2)$$
$$= 2 + 3^a(8n-4) + 9^a(6n^2-8n+2).$$

We obtain the following results by Theorem 7.

Corollary 7.1. The reduced second Zagreb index of $RHOX_n$ is
$RM_2(RHOX_n) = 54n^2 - 48n + 8$.

Proof: Put $a = 1$ in equation (6), we get the desired result.

Corollary 7.2. The reduced second hyper-Zagreb index of $RHOX_n$ is
$RHM_2(RHOX_n) = 486n^2 - 567n + 128$.

Proof: Put $a = 2$ in equation (6), we get the desired result.

Theorem 8. The reduced second Zagreb polynomial of $RHOX_n$ is
$RM_2(RHOX_n, x) = 2x^1 + (8n-4)x^3 + (6n^2-8n+2)x^9$.

Proof: Let $G=RHOX_n$ be the graph of rhombus oxide network. By using equation (1) and Table 3, we have

$$RM_2(RHOX_n, x) = \sum_{uv\in E(G)} x^{(d_G(u)-1)(d_G(v)-1)}$$
$$= 2x^{(2-1)(2-1)} + (8n-4)x^{(2-1)(4-1)} + (6n^2-8n+2)x^{(4-1)(4-1)}$$
$$= 2x^1 + (8n-4)x^3 + (6n^2-8n+2)x^9.$$

Theorem 9. The reduced second hyper Zagreb polynomial of $RHOX_n$ is
$RHM_2(RHOX_n, x) = 2x^1 + (8n-4)x^9 + (6n^2-8n+2)x^{81}$.

Proof: Let $G = RHOX_n$ be the graph of rhombus oxide network. By using equation (3) and Table 3, we derive

$$RHM_2(RHOX_n, x) = \sum_{uv\in E(G)} x^{\left[(d_G(u)-1)(d_G(v)-1)\right]^2}$$
$$= 2x^{[(2-1)(2-1)]^2} + (8n-4)x^{[(2-1)(4-1)]^2} + (6n^2-8n+2)x^{[(4-1)(4-1)]^2}$$
$$= 2x^1 + (8n-4)x^9 + (6n^2-8n+2)x^{81}.$$

Honeycomb Networks

If we recursively use hexagonal tiling in a particular pattern, honeycomb networks are formed. These networks are very useful in Chemistry and also in Computer Graphics. A honeycomb network of dimension n is denoted by HC_n, where n is the number of hexagons between central and boundary hexagon. A 4-dimensional honeycomb network is shown in Figure 4.

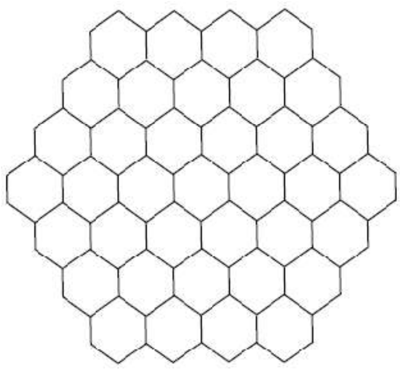

Figure 4. Honeycomb network of dimension 4

Let G be the graph of a honeycomb network HC_n with $6n^2$ vertices and $9n^2–3n$ edges. In HC_n, by calculation, there are three types of edges based on the degree of end vertices of each edge as given in Table 4.

Table 4. Edge partition of HC_n

$d_G(u), d_G(v)\backslash uv \in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	6	$12n – 12$	$9n^2 – 15n + 6$

Theorem 10. The general reduced second Zagreb index of a honeycomb network HC_n is

$$RM_2^a(HC_n) = 6 + 2^a(12n – 12) + 4^a(9n^2 – 15n + 6). \qquad \ldots\ldots\ldots\ldots(7)$$

Proof: Let $G=HC_n$ be the graph of honeycomb network. By using equation (3) and Table 4, we deduce

$$RM_2^a(HC_n) = \sum_{uv\in E(G)} \left[(d_G(u)-1)(d_G(v)-1)\right]^a$$

$$= [(2–1)(2–1)]^a 6 + [(2–1)(3–1)]^a(12n – 12) + [(3–1)(3–1)]^a (9n^2 – 15n + 6)$$

$$= 6 + 2^a(12n – 12) + 4^a(9n^2 – 15n + 6).$$

We obtain the following results by using Theorem 10.

Corollary 10.1. The reduced second Zagreb index of HC_n is

$$RM_2(HC_n) = 36n^2 – 36n + 6.$$

Proof: Put $a = 1$ in equation (7), we get the desired result.

Corollary 10.2. The reduced second hyper-Zagreb index of HC_n is

$$RHM_2(HC_n) = 144n^2 – 192n + 54.$$

Proof: Put $a = 2$ in equation (7), we get the desired result.

Theorem 11. The reduced second Zagreb polynomial of HC_n is

$$RM_2(HC_n, x) = 6x^1 + (12n – 12)\, x^2 + (9n^2 – 15n + 6)\, x^4.$$

Proof: Let $G = HC_n$ be the graph of honeycomb network. By using equation (1) and Table 4, we obtain

$$RM_2(HC_n, x) = \sum_{uv\in E(G)} x^{(d_G(u)-1)(d_G(v)-1)}$$

$$= 6x^{(2–1)(2–1)} + (12n – 12)x^{(2–1)(3–1)} + (9n^2 – 15n + 2)x^{(3–1)(3–1)}$$

$$= 6x^1 + (12n – 12)x^2 + (9n^2 – 15n + 6)\, x^4.$$

Theorem 12. The reduced second hyper Zagreb polynomial of HC_n is

$$RHM_2(HC_n, x) = 6x^1 + (12n – 12)\, x^4 + (9n^2 – 15n + 6)\, x^{16}.$$

Proof: Let $G = HC_n$ be the graph of honeycomb network. By using equation (2) and Table 4, we derive

$$RHM_2(HC_n, x) = \sum_{uv\in E(G)} x^{\left[(d_G(u)-1)(d_G(v)-1)\right]^2}$$

$$= 6x^{[(2-1)(2-1)]^2} + (12n - 12)x^{[(2-1)(3-1)]^2} + (9n^2 - 15n + 6)x^{[(3-1)(3-1)]^2}$$

$$= 6x^1 + (12n – 12)\, x^4 + (9n^2 – 15n + 6)\, x^{16}.$$

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