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RESEARCH ARTICLE

LEAP INDICES OF GRAPHS

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ABSTRACT

We propose the modified first leap index, leap inverse degree, leap zeroth order index, the general first leap Zagreb index of a graph. Furthermore, we compute the modified first leap index, F-leap index, leap zeroth order index and general first leap Zagreb index of certain wheel related graphs such as wheels, gear graphs, helm graphs, flower graphs and sunflower graphs.

Key words: Modified First Leap Index, F-Leap Index, Wheel, Helm Graph, Flower Graph.

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INTRODUCTION

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The distance $d(u, v)$ between any two vertices u and v of G is the length of a shortest path connecting them. For a positive integer k, the open k-neighborhood $N_k(v)$ of a vertex v in G is defined as $N_k(v/G) = \{u \in V(G) : d(u, v) = k\}$. The k-distance degree $d_k(v)$ of v in G is defined as the number of k neighbors of v in G. The degree $d(v)$ of a vertex v is the number of edges incident to v. We refer to [Kulli, 2012] for undefined term and notation.
The total 2-distance degree of a graph G is defined as

$$T_2(G) = \sum_{u \in V(G)} d_2(u).$$

The first leap Zagreb index of G is defined as

$$LM_1(G) = \sum_{u \in V(G)} d_2^2(u).$$

This index was introduced in [Naji, 2017].

We propose the following the leap Zagreb indices. The modified first leap index of G is defined as

$$^{m}LM_1(G) = \sum_{u \in V(G)} \frac{1}{d_2^2(u)}.$$

The F-leap index of G is defined as

$$FL(G) = \sum_{u \in V(G)} d_2^3(u).$$

The leap inverse degree of G is defined as

$$LID(G)=\sum_{u\in V(G)}\frac{1}{d_2(u)}.$$

The leap zeroth order index of G is defined as

$$LZ(G)=\sum_{u\in V(G)}\frac{1}{\sqrt{d_2(u)}}.$$

The general first leap Zagreb index of G is defined as

$$LM_1^a(G)=\sum_{u\in V(G)}d_2^a(u) \tag{1}$$

where a is a real number.

Furthermore, we propose the first leap polynomial and F-leap polynomial of G as

$$LM_1(G,x)=\sum_{u\in V(G)}x^{d_2^2(u)} \tag{2}$$

$$FL(G,x)=\sum_{u\in V(G)}x^{d_2^3(u)} \tag{3}$$

Recently, some leap indices were studied such as leap hyper Zagreb indices [Kulli, 2018], minus leap and square leap indices [Kulli, 2018], F-leap indices [Kulli, 2018], sum connectivity leap and geometric-arithmetic leap indices [Kulli], product connectivity leap index and ABC leap index [Kulli, 2018], multiplicative leap and multiplicative hyper leap indices [8], augmented leap index [Kulli]. Very recently, some new polynomials were introduced and studied, for example, in [Kulli, 2017 and 2018]. In this paper, wheel graphs and wheel type graphs are considered, see [Shiladhar, 2018]. The modified first leap index, leap zeroth order index, F-leap index, general first leap Zagreb index of wheel, gear, helm, flower, sunflower graphs are computed.

RESULTS FOR WHEELS

The wheel W_{n+1} is defined to be the graph K_1+C_n, $n\geq 3$. The wheel W_{n+1} has $n+1$ vertices and $2n$ edges. The vertices of C_n are called rim vertices and the vertex of K_1 is called apex.

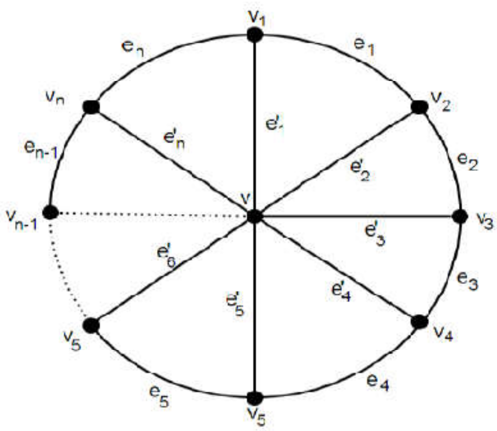

Figure 1. Wheel W_{n+1}

Let $G = W_{n+1}$. There are two types of the 2-distance degree of vertices in W_{n+1} as follows:

$V_1=\{ u \in V(G) \mid d_2(u) = 0\}$, $|V_1| = 1$.

$V_2= \{u \in V(G)| \ d_2(u) = n-3\}$ $|V_2| = n$.

Theorem 1. The general first leap Zagreb index of W_{n+1} is

$$LM_1^a(W_{n+1})=n(n-3)^a. \tag{4}$$

Proof: Let W_{n+1} be a wheel with $n \geq 3$ vertices. From equation (1) and by cardinalities of the 2-distance degree of vertex partition of W_{n+1}, we have

$$LM_1^a(W_{n+1}) = \sum_{u \in V(G)} d_2^a(u)$$
$$= \sum_{u \in V_1} d_2^a(u) + \sum_{u \in V_2} d_2^a(u)$$
$$= 1 \times 0^a + n(n-3)^a$$
$$= n(n-3)^a.$$

From Theorem 1, we establish the following results.

Corollary 1.1. [24] The first leap Zagreb index of W_{n+1} is

$$LM_1(W_{n+1}) = n(n-3)^2.$$

Corollary 1.2. The modified the first leap Zagreb index of W_{n+1} is

$${}^m LM_1(W_{n+1}) = \frac{n}{(n-3)^2}.$$

Corollary 1.3. The F-leap index of W_{n+1} is

$$FL(W_{n+1}) = n(n-3)^3.$$

Corollary 1.4. The leap inverse degree of W_{n+1} is

$$LID(W_{n+1}) = \frac{n}{n-3}.$$

Corollary 1.5. The leap zeroth-order index of W_{n+1} is

$$LZ(W_{n+1}) = \frac{n}{\sqrt{n-3}}.$$

Put $a = 2, -2, 3, -1, -½$ in equation (4), we get the above results respectively.

Theorem 2. Let W_{n+1} be a wheel with $n+1$ vertices, $n \geq 3$. Then

i) $LM_1(W_{n+1}, x) = x^0 + nx^{(n-3)^2}$.

ii) $FL(W_{n+1}, x) = x^0 + nx^{(n-3)^3}$.

Proof: (i) From equation (2) and by cardinalities of the 2-distance degree of vertex partition, we obtain

$$LM_1(W_{n+1}, x) = \sum_{u \in V(G)} x^{d_2^2(u)}$$
$$= x^0 + nx^{(n-3)^2}.$$

From equation (3) and by cardinalities of the 2-distance degree of vertex partition, we have

$$FL(W_{n+1}, x) = \sum_{u \in V(G)} x^{d_2^3(u)}$$
$$= x^0 + nx^{(n-3)^3}.$$

RESULTS FOR GEAR GRAPHS

A bipartite wheel graph is a graph obtained from W_{n+1} by adding a vertex between each pair of adjacent rim vertices and it is denoted by G_n, also called as a gear graph. Clearly $|V(G_n)|=2n+1$ and $|E(G_n)| = 3n$.

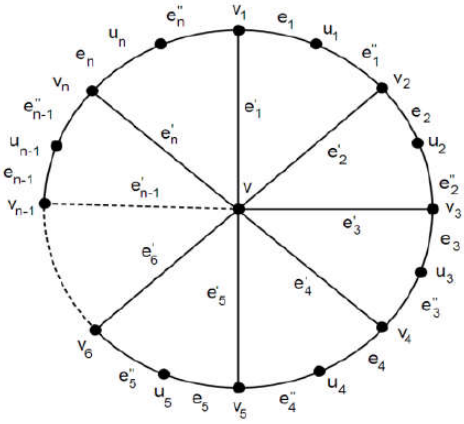

Figure 2. Gear graph G_n

There are three types of the 2-distance degree of vertices in G_n as follows:

$V_1 = \{u \in V(G_n) | \, d_2(u) = n\}$, $|V_1| = 1$.
$V_2 = \{u \in V(G_n) \, | \, d_2(u) = n - 1\}$, $|V_2| = n$.
$V_3 = \{u \in V(G_n) | \, d_2(u) = 3\}$, $|V_3| = n$.

Theorem 3. The general first leap Zagreb index of G_n is

$$LM_1^a(G_n) = n^a + n(n-1)^a + 3^a n. \tag{5}$$

Proof: Let G_n be a gear graph with $2n+1$ vertices, $n \geq 3$. From equation (1) and by cardinalities of the 2-distance degree of vertex partition of G_n, we obtain

$$LM_1^a(G_n) = \sum_{u \in V(G_n)} d_2^a(u)$$
$$= \sum_{u \in V_1} d_2^a(u) + \sum_{u \in V_2} d_2^a(u) + \sum_{u \in V_3} d_2^a(u)$$
$$= n^a + n(n-1)^a + n \times 3^a.$$

From Theorem 3, we obtain the following results.

Corollary 3.1. [24] The first leap Zagreb index of G_n is

$$LM_1(G_n) = n(n^2 - n + 10).$$

Corollary 3.2. The modified the first leap Zagreb index of G_n is

$$^m LM_1(G_n) = \frac{1}{n^2} + \frac{n}{(n-1)^2} + \frac{n}{9}.$$

Corollary 3.3. The F-leap index of G_n is

$$FL(G_n) = n(n^3 - 2n^2 + 3n + 26).$$

Corollary 3.4. The leap inverse degree of G_n is

$$LID(G_n) = \frac{1}{n} + \frac{n}{n-1} + \frac{n}{3}.$$

Corollary 3.5. The leap zeroth-order index of G_n is

$$LZ(G_n) = \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n-1}} + \frac{n}{\sqrt{3}}.$$

Put $a = 2, -2, 3, -1, -½$ in equation (5), we obtain the above results respectively.

Theorem 4. Let G_n be a gear graph with $2n+1$ vertices, $n\geq 3$. Then

i) $LM_1(G_n, x) = x^{n^2} + nx^{(n-1)^2} + nx^9$.

ii) $FL(G_n, x) = x^{n^3} + nx^{(n-1)^3} + nx^{27}$.

Proof: (i) From equation (2) and by cardinalities of the 2-distance degree of vertex partition, we have

$$LM_1(G_n, x) = \sum_{u\in V(G)} x^{d_2^2(u)}$$
$$= x^{n^2} + nx^{(n-1)^2} + nx^9.$$

From equation (3) and by cardinalities of the 2-distance degree of vertex partition, we deduce

$$FL(G_n, x) = \sum_{u\in V(G)} x^{d_2^3(u)}$$
$$= x^{n^3} + nx^{(n-1)^3} + nx^{27}.$$

RESULTS FOR HELM GRAPHS

A helm graph H_n is a graph obtained from W_{n+1} by attaching an end edge to each rim vertex. Clearly $|V(H_n)|=2n+1$ and $|E(H_n)|=3n$.

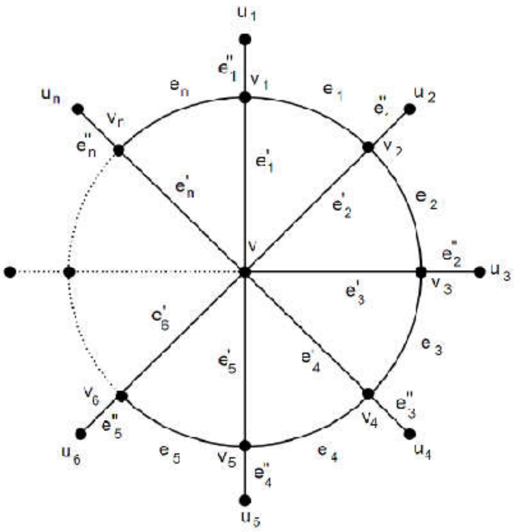

Figure 3. Helm graph H_n

There are three types of the 2-distance degree of vertices in H_n as follows.

$V_1 = \{u\in V(H_n) \mid d_2(u) = n\}$	$\lvert V_1\rvert = 1.$
$V_2 = \{u\in V(H_n) \mid d_2(u) = n-1\}$	$\lvert V_2\rvert = n.$
$V_3 = \{u\in V(H_n) \mid d_2(u) = 3\}$	$\lvert V_3\rvert = n.$

The 2-distance degree of vertices of H_n and G_n are same. Therefore we have the following results.

Theorem 5.

(i) $LM_1^a(H_n) = LM_1^a(G_n) = n^a + n(n-1)^a + 3^a n.$

(ii) $LM_1(H_n) = LM_1(G_n) = n(n^2 - n + 10)$, see [24].

(iii) ${}^m LM_1(H_n) = {}^m LM_1(G_n) = \frac{1}{n^2} + \frac{n}{(n-1)^2} + \frac{n}{9}.$

(iv) $FL(H_n) = FL(G_n) = n(n^3 - 2n^2 + 3n + 26).$

(v) $LID(H_n) = LID(G_n) = \frac{1}{n} + \frac{n}{n-1} + \frac{n}{3}.$

(vi) $LZ(H_n) = LZ(G_n) = \frac{1}{\sqrt{n}} + \frac{n}{\sqrt{n-1}} + \frac{n}{\sqrt{3}}.$

(vii) $LM_1(H_n, x) = LM_1(G_n, x) = x^{n^2} + nx^{(n-1)^2} + nx^9.$

(viii) $FL(H_n, x) = FL(G_n, x) = x^{n^3} + nx^{(n-1)^3} + nx^{27}.$

RESULTS FOR FLOWER GRAPHS

The graph Fl_n is a flower graph obtained from a helm graph by joining an end vertex to the apex of the helm graph. The graph Fl_n has $2n+1$ vertices and $4n$ edges.

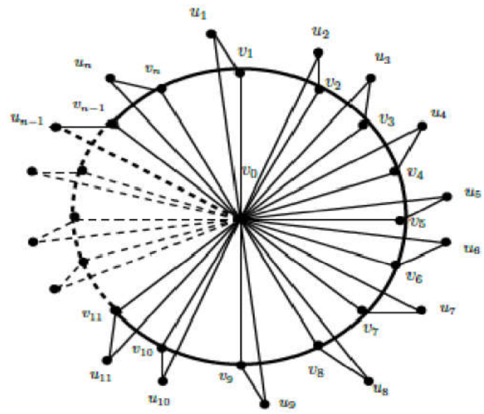

Figure 4. Flower graph Fl_n.

There are three types of 2 distance degree of vertices in Fl_n as follows:

$V_1 = \{u \in V(Fl_n) \mid d_2(u) = 0\}$	$\lvert V_1\rvert = 1.$
$V_2 = \{u \in V(Fl_n) \mid d_2(u) = n-5\}$	$\lvert V_2\rvert = n.$
$V_3 = \{u \in V(Fl_n) \mid d_2(u) = n-2\}$	$\lvert V_3\rvert = n.$

Theorem 6. The general first leap Zagreb index of Fl_n is

$$LM_1^a(Fl_n) = n(n-5)^a + n(n-2)^a. \tag{6}$$

Proof: Let Fl_n be a flower graph with $2n+1$ vertices, $n \geq 3$. From equation (1) and by cardinalities of the 2-distance degree of vertex partition of Fl_n, we have

$$LM_1^a(Fl_n) = \sum_{u \in V(Fl_n)} d_2^a(u)$$
$$= \sum_{u \in V_1} d_2^a(u) + \sum_{u \in V_2} d_2^a(u) + \sum_{u \in V_3} d_2^a(u)$$
$$= 0 + n(n-5)^a + n(n-2)^a.$$
$$= n(n-5)^a + n(n-2)^a.$$

From Theorem 6, we have the following results.

Corollary 6.1. [24] The first leap Zagreb index of Fl_n is

$$LM_1(Fl_n) = n(2n^2 - 14n + 29).$$

Corollary 6.2. The modified the first leap Zagreb index of Fl_n is

$${}^{m}LM_1(Fl_n) = \frac{n}{(n-5)^2} + \frac{n}{(n-2)^2}.$$

Corollary 6.3. The F-leap index of Fl_n is

FL(Fln) = n(n-5)3 + n(n-2)2

Corollary 6.4. The leap inverse degree of Fl_n is

$$LID(Fl_n) = \frac{n}{(n-5)} + \frac{n}{(n-2)}.$$

Corollary 6.5. The leap zeroth order index of Fl_n is

$$LZ(Fl_n) = \frac{n}{\sqrt{n-5}} + \frac{n}{\sqrt{n-2}}.$$

Put $a = 2, -2, 3, -1, -½$ in equation (6), we get the above results respectively.

Theorem 7. Let Fl_n be a flower graph with $2n+1$ vertices. Then

i) $LM_1(Fl_n, x) = x^0 + nx^{(n-5)^2} + nx^{(n-2)^2}$.

ii) $FL(Fl_n, x) = x^0 + nx^{(n-5)^3} + nx^{(n-2)^3}$.

Proof: (i) From equation (2) and by cardinalities of the 2-distance degree of vertex partition, we derive

$$\begin{aligned} LM_1(Fl_n, x) &= \sum_{u \in V(G)} x^{d_2^2(u)} \\ &= x^0 + nx^{(n-5)^2} + nx^{(n-2)^2}. \end{aligned}$$

From equation (3) and by cardinalities of the 2-distance degree of vertex partition, we deduce

$$\begin{aligned} FL(Fl_n, x) &= \sum_{u \in V(G)} x^{d_2^3(u)} \\ &= x^0 + nx^{(n-5)^3} + nx^{(n-2)^3}. \end{aligned}$$

RESULTS FOR SUNFLOWER GRAPHS

The graph Sf_n is a sunflower graph obtained from Fl_n by attaching n end edges to the apex vertex. The graph Sf_n has $3n+1$ vertices and $5n$ edges.

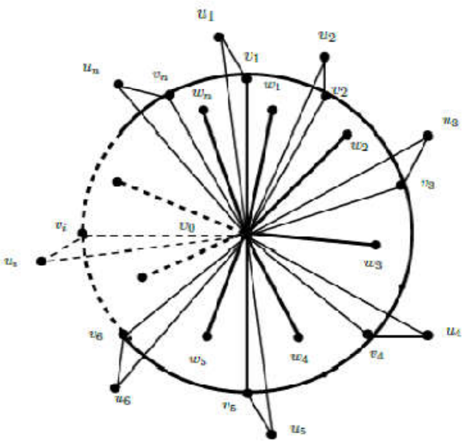

Figure 5. Sunflower graph Sf_n

There are four types of the 2 distance degree of vertices in Sf_n as follows.

$V_1 = \{u \in V(Sf_n) \mid d_2(u) = 0\}$	$\lvert V_1 \rvert = 1$
$V_2 = \{u \in V(Sf_n) \mid d_2(u) = 3n - 4\}$	$\lvert V_2 \rvert = n$
$V_3 = \{u \in V(Sf_n) \mid d_2(u) = 3n - 2\}$	$\lvert V_3 \rvert = n$
$V_4 = \{u \in V(Sf_n) \mid d_2(u) = 3n - 1\}$	$\lvert V_4 \rvert = n$.

Theorem 8. The general first leap Zagreb index of Sf_n is

$$LM_1^a(Sf_n) = n(3n-4)^a + n(3n-2)^a + n(3n-1)^a \tag{7}$$

Proof: Let Sf_n be a sunflower graph with $2n+1$ vertices. From equation (1) and by cardinalities of the 2-distance degree of vertex partition of Sf_n, we have

$$\begin{aligned} LM_1^a(Sf_n) &= \sum_{u \in V(Sf_n)} d_2^a(u) \\ &= \sum_{u \in V_1} d_2^a(u) + \sum_{u \in V_2} d_2^a(u) + \sum_{u \in V_3} d_2^a(u) + \sum_{u \in V_4} d_2^a(u) \\ &= 0 + n(3n-4)^a + n(3n-2)^a + n(3n-1)^a. \\ &= n(3n-4)^a + n(3n-2)^a + n(3n-1)^a. \end{aligned}$$

From Theorem 8, we obtain the following results.

Corollary 8.1. [24] The first leap Zagreb index of Sf_n is

$$LM_1(Sf_n) = 3n(9n^2 - 14n + 7)$$

Corollary 8.2. The modified the first leap Zagreb index of Sf_n is

$$^{m}LM_n(Sf_n) = \frac{n}{(3n-4)^2} + \frac{n}{(3n-2)^2} + \frac{n}{(3n-1)^2}.$$

Corollary 8.3. The F-leap index of Sf_n is

$$FL(Sf_n) = n(3n-4)^3 + n(3n-2)^3 + n(3n-1)^3.$$

Corollary 8.4. The leap inverse degree of Sf_n is

$$LID(Sf_n) = \frac{n}{3n-4} + \frac{n}{3n-2} + \frac{n}{3n-1}.$$

Corollary 8.5. The leap zeroth-order index of Sf_n is

$$LZ(Sf_n) = \frac{n}{\sqrt{3n-4}} + \frac{n}{\sqrt{3n-2}} + \frac{n}{\sqrt{3n-1}}.$$

Put $a = 2, -2, 3, -1, -1, -\frac{1}{2}$ is equation (7), we obtain the above results respectively.

Theorem 9. Let Sf_n be a sunflower graph with $3n + 1$ vertices. Then

i) $LM_1(Sf_n, x) = x^0 + nx^{(3n-4)^2} + nx^{(3n-2)^2} + nx^{(3n-1)^2}.$

ii) $FL(Sf_n, x) = x^0 + nx^{(3n-4)^3} + nx^{(3n-2)^3} + nx^{(3n-1)^3}.$

Proof: (i) From equation (2) and by cardinalities of the 2-distance degree of vertex partition, we derive

$$\begin{aligned} LM_1(Sf_n, x) &= \sum_{u \in V(Sf_n)} x^{d_2^2(u)} \\ &= x^0 + nx^{(3n-4)^2} + nx^{(3n-2)^2} + nx^{(3n-1)^2}. \end{aligned}$$

(ii) From equation (3) and by cardinalities of the 2-distance degree of vertex partition, we deduce

$$\begin{aligned} FL(Sf_n, x) &= \sum_{u \in V(Sf_n)} x^{d_2^3(u)} \\ &= x^0 + nx^{(3n-4)^3} + nx^{(3n-2)^3} + nx^{(3n-1)^3}. \end{aligned}$$

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